Measuring the Average Thickness of a Plate Using a Bayesian Method and Free Vibration Data

E. Z. MOORE, K. D. MURPHY and J. M. NICHOLS

Abstract

In this paper a model-based approach is taken to identify certain geometric and/or material parameters for a structure. Specifically, the focus is on determining the average thickness of a thin, fully-clamped plate undergoing free vibration. The methodology is described and then implemented in an experimental setting using the measured free response of a plate. The practical SHM scenario driving this investigation is corrosion, since the extent of corrosion damage is often described in terms of the effective thickness of the plate, i.e. how much of the plate's thickness is still structurally intact. Data are gathered from three resistive strain gages, placed at arbitrary locations and orientations. Using the experimental response and a finite element model, a Bayesian approach is taken to estimate the plate thickness, *h*. This thickness could then be used to infer the extent of the damage, had corrosion been present. The results show that even with limited, noisy vibration data valuable information regarding the geometry and material characteristics of a plate can be successfully estimated.

1 INTRODUCTION

Research into the effects of corrosion on the structural integrity of a component has been an active field for many years. Such work has been motivated by a variety of applications, including ocean going vessels, civil engineering structures (e.g., bridges), etc. The extent of the corrosion damage is dependent on the material used in the structure and on environmental factors. For maximum accuracy, the description of the damage should be treated as spatially stochastic [1]. This unfortunately requires significant knowledge of the plate and how the environment factors into the system. For example, the extent to which a bridge structure is submerged in sea water might

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14. ABSTRACT
In this paper a model-based approach is taken to identify certain geometric and/or material parameters for a structure. Specifically, the focus is on determining the average thickness of a thin, fully-clamped plate undergoing free vibration. The methodology is described and then implemented in an experimental setting using the measured free response of a plate. The practical SHM scenario driving this investigation is corrosion, since the extent of corrosion damage is often described in terms of the effective thickness of the plate, i.e. how much of the plates thickness is still structurally intact. Data are gathered from three resistive strain gages, placed at arbitrary locations and orientations. Using the experimental response and a finite element model, a Bayesian approach is taken to estimate the plate thickness, h. This thickness could then be used to infer the extent of the damage, had corrosion been present. The results show that even with limited, noisy vibration data valuable information regarding the geometry and material characteristics of a plate can be successfully estimated.

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vary with the tides, making the exposure time-varying. Requiring this level of knowledge of a real-world system is, to say the least, impractical. Others have *predicted* (i.e., not measured) an effective thickness using service time and tabulated constants that account for environmental conditions [2]. This effective thickness is then used to predict remaining life. An alternate approach compares the strength of a uniformly and stochastically corroded plates under different loading conditions [3, 4, 5]. It has been shown that the uniform effective thickness approximation produces conservative estimate for plate strength in both bending and compression. While the effective thickness approach is useful/conservative in estimating remaining life, it is not overly practical. Why? Because there is not a simple and reliable technique for obtaining the effective thickness in-situ; direct measurements are either difficult or impossible. One is left to the guesswork and legwork described in Reference [2].

The present paper does not focus on corrosion, per se. Instead, it lays out a consistent (and simple) procedure for determining from experimental data the effective thickness of a plate, regardless of its condition or accessibility. But the approach is more flexible and broadly applicable than this. It can actually be used to identify *any* arbitrary geometric parameter or material property that can be incorporated into a mathematical/numerical (usually physics-based) model for use in the Bayesian methodology. See, for example, the crack and delamination detection examples provided in References [6, 7, 7]. But the Bayesian approach does not stop at giving estimates of the desired parameter(s). It also gives an actual probability distribution for the parameter, such that you can obtain meaningful credible intervals (the Bayesian version of a confidence interval); one finds an estimate of the parameter *and* obtains a measure of how certain that answer is. And this has been expanded to a incorporate global searches, using what is called a population-based Bayesian approach [8]. The "pop" approach opens the door to a more thorough search of the space of possible solutions and may lead to multi-modal parameter distributions.

In the following section, a brief description of the methodology is described, including a summary of the finite element model used. This is followed by a description of the experimental setup, the sensors, the excitation, and the data itself. This is followed by a few results and some general conclusions and future work. It is found that this approach provides a straight forward and reliable means to determine effectively the predicted thickness of the plate and its probability distribution.

2 THE BAYESIAN APPROACH

The following is a brief discussion of the Bayesian and population-based Bayesian approaches for determining the unknown parameters and their probability distributions. For a thorough discussion of these techniques, see Reference [7].

The parameter identification process requires three things. First, the free response of the structure under consideration must be measured. Second, a dynamic model of the structure, containing the unknown parameter (here the thickness, h), must be developed; this model should be capable of predicting the free response, such that a direct comparison to the experimental response can be made. Third, the Bayesian methodology has to be implemented; this take the two signals and produces the parameter estimate and its distribution.

2.1 The Standard Bayesian/MCMC Procedure

The experimental and model predicted responses are recorded in the vectors $\mathbf{y}$ and $\mathbf{x}(\theta)$, respectively. θ is a vector containing the unknown (to be determined) parameters. In this case, θ is simply the scalar h. The objective is to use the observations $\mathbf{y}$ to estimate the probability distribution for the thickness. In Bayesian terms, this is referred to as the posterior: $p_{\theta_i}(\theta_i|y)$. This is read as the probability of θ given $\mathbf{y}$ or, alternatively, the probability of h conditioned on $\mathbf{y}$.

Another key concept used in the Bayesian approach is the prior distribution: $p_\pi(\boldsymbol{\theta},\sigma^2)$. This probability incorporates any a-priori knowledge one might have about the system. For example, if one hundred similar structures had been examined and their thicknesses were all approximately the same (h_{ave}), this constitutes valuable prior knowledge. The *initial* parameter search could then be directed toward that value by choosing a prior distribution with a single peak at h_{ave}. If no information is known a-priori, then a uniform prior distribution could be assumed.

Relating these two distributions (prior to posterior) is the *likelihood*. It represents the probability of the data being recorded from a test given a presumed plate thickness. Using the same notational form, the likelihood is expressed as: $p_L(\mathbf{y}|\boldsymbol{\theta})$. The likelihood is formed by considering the uncertainty in our observations. Consider, for example, describing the measured response at sensor "r" at discrete time "n" as $\mathbf{y} \equiv y_{rn}$. Similarly, the model response is denoted $\mathbf{x} \equiv x_{rn}$. These two differ by some amount of error - both modeling error and experimental noise. Thus, one may write:

$$y_{rn} = x_{rn}(\boldsymbol{\theta}) + \eta_{rn}, \quad n = 1,2,3....N, \quad r = 1,\cdots,M \tag{1}$$

where N is number of recorded instances in the time series, M is the number of sensors, and η_{rn} is the error. Both sources of error are assumed to be Gaussian. With this assumption, the probability that the data $\mathbf{y}$ was produced by the model, under the assumed parameters ($\boldsymbol{\theta}$) and the (unknown) noise variance σ^2, is:

$$p_L(\mathbf{y}|\boldsymbol{\theta},\sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{MN}{2}}} \exp\left[-\frac{1}{2\sigma^2}\sum_{r=1}^{M}\sum_{n=1}^{N}(y_{rn} - x_{rn}(\boldsymbol{\theta}))^2\right]. \tag{2}$$

This is the likelihood function. Equation (2) is clearly governed by the sum of the squared error between the experimental data and the predicted model response: $\sum\sum(y_{rn} - x_{rn}(\boldsymbol{\theta}))^2$.

Relating these three distributions (the posterior [our objective], the prior, and the likelihood) is accomplished via Bayes' theorem:

$$p_\Theta(\boldsymbol{\theta},\sigma^2|y) = \frac{p_J(\boldsymbol{\theta},\sigma^2,\mathbf{y})}{p_D(\mathbf{y})} = \frac{p_L(\mathbf{y}|\boldsymbol{\theta},\sigma^2)p_\pi(\boldsymbol{\theta},\sigma^2)}{p_D(\mathbf{y})}. \tag{3}$$

The term on the left is, in fact, the desired posterior. The prior and the likelihood are in the numerator of the right side. And the denominator is a normalizing quantity, which is very difficult to evaluate. So instead of directly calculating the posterior from Equation (3), the Markov-chain Monte Carlo (MCMC) approach is taken to evaluate the posterior [9], [10].

In short, the MCMC process is as follows. Using the prior, an initial guess for the plate thickness h is generated. This parameter is used in the model to generate the

response $x_{rn}(\boldsymbol{\theta})$. This, along with the experimental signal y_{rn}, are used to generate the likelihood, Equation (2). The parameter (h) is perturbed and the process is carried out again. A Bernoulli trial (with an approximately 40% acceptance rate) is used to either keep the new value or discard it. Either way, the parameter is perturbed again and the process repeated. This creates the MCMC chain, whose distribution is (exactly) the desired posterior.

2.2 Population-Based MCMC

The MCMC method provides a good description of the posterior if there is a single minima. However, for more complex problems with multiple minima, this approach may just explore one local minima. In the process, the generated posterior does not provide a good estimate of the desired parameter (h) nor does it reflect the complexity of the problem. In the frequentist framework, methods such as genetic algorithms and simulated annealing are commonly used to find global minima. To explore more complex spaces multiple chains may be run simultaneously [11, 12].

Without going into the mathematical details, the pop-based MCMC involves creating k chains, each with different initial guesses (of h) spread throughout the space. The first chain is of the actual posterior. Subsequent chains are raised to successively higher powers, smoothing the distributions. These chains all move forward with each iteration and, with a 50% probability, the chains are randomly *swapped*. This way, all of the chains are informing one another as to the presence of multiple peaks within the distribution. For a more detailed discussion of population based Bayesian methods see Reference [12].

3 MECHANICS MODEL

Regardless of the type of model used, many estimation methods require repeatedly evaluating the model with different parameter combinations. So while accuracy and computational speed both matter, speed is the dominant factor. The finite element method is used to model the plate, with the number of degrees of freedom being kept to a minimum. A 20 by 16 element quadrilateral mesh is used. Despite the coarseness of the mesh, the model still matches the theoretical predictions of the first and second natural frequencies for a cracked plate to within 0.5%. The model is classically damped, with all modes sharing the same damping ratio of 0.003. For a detailed discussion of this model see Reference [7].

The unknown model parameter of interest is the thickness of the plate, though in theory any geometric or material property could be placed in the vector θ. There is an additional “nuisance” structural parameter, whose use is necessitated by the differences between perfect theoretical models and physical reality. This additional parameter is a lumped rotational stiffness added to the nodes on the perimeter of the plate, which allows the finite element model to represent the rigid-seeming, yet imperfect, boundary condition used in the experiment. While the plate is well clamped, as described in section 4, it is not *perfectly* clamped. Instead of a fully clamped condition, and eliminating rows and columns from the global stiffness matrix, the plate is considered simply supported with a large lumped rotational stiffness applied around

Normalized Dim.	Value
Width (L_x/L_y)	1.25
Length (L_y/L_y)	1
Thickness (h/L_y)	0.0026

Figure 1 & Table 1: The experimental test setup and nondimensional plate geometry

the perimeter of the plate. A constant is added to the edge rotational degrees of freedom elements of the global stiffness matrix. For each node, the constant is scaled by the edge length of each finite element so as to produce a comparable result between meshes of different element density. The output of this model is the rotational deflection of the plate at the sensor locations of the experimental plate. Strain is calculated from these rotations using central differences.

The actual time response is determined via modal analysis. The FEM model is used to determine the natural frequencies and mode shapes. The analytic modal solution is then generated from these numerical eigen-results.

In its current form, the model is constructed with the implicit assumption that the corrosion is uniform. A more detailed model that included details of the pitting would require too many variables. A large number of variables would greatly increase the solution time, and it is likely that different pitting patterns could have very similar impacts on the vibration response, making it difficult to distinguish between them without many more sensors.

4 EXPERIMENTAL PROCEDURE

The test specimen was a 0.76m by 0.60m (30” by 24”, 5:4 aspect ratio), 1.55mm (1/16”) thick 6061-T6 aluminum plate. The density was calculated using a digital scale and a vernier caliper. Normalized dimensions are given in Table 1.

The plate was clamped in a bolted fixture shown in Figure 1. The values of Young’s modulus and Poisson’s ratio used by the model were determined via tension tests following the ASTM E8 standard.

4.1 EXCITATION

The plate was excited from rest by the impact of a simple rubber-tipped mallet. The impact point was selected to be an arbitrary point along the diagonal of the plate. There are two phenomena not captured by the model that are dominant immediately after impact. The first is nonlinearity of the response due to the large transverse deflection experienced. For deflections larger than about one plate thickness membrane stresses are significant, whereas the finite element model includes only the effects of

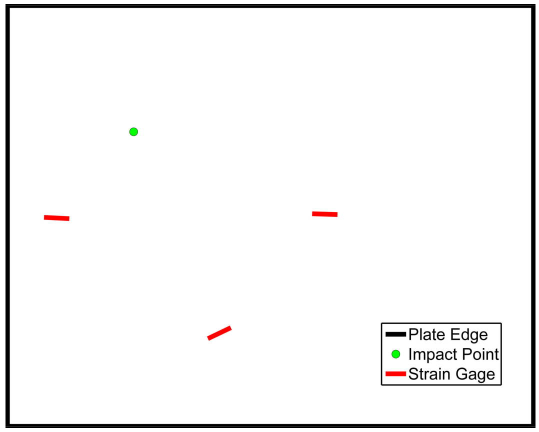

Figure 2: The arbitrarily selected locations and orientations of the three strain gages (short red lines) and the impact location (circle)

bending. The second unmodeled phenomenon is the power in non-modal frequencies transmitted by impact. This "ringing" in the response dissipated quickly, but must not be compared to the model prediction by the MCMC process, as the behavior can not be captured by the simple model. A more complex model may be used, but at the cost of computational speed. Once these two effect subside, free bending vibration dominates the response. Hence, the first part of the response ($t < t_o$) must be omitted.

The initial conditions of the plate are needed to compute the amplitudes and phases of the various terms in the solution. Unfortunately, at t_o - the beginning of the signal that was retained - the initial conditions are unknown. Using the strain and the strain rate (found via central difference) at t_o, the unknown amplitudes and phases were determined numerically.

4.2 SENSORS

Three single-axis Vishay MicroMeasurements model EA-13-125AD-120 strain gages were located in arbitrary positions and orientations as shown in Figure 2. The signals were sampled at a frequency of 25kHz. In the case of a heavily corroded plate, accelerometers may be a better choice because they are easier to apply to uneven surfaces, but the type of data produced is similar, and so the method used here would be unaffected by the change.

The signals were filtered with a passband filter between the frequencies of 13Hz and 66Hz. These frequencies were chosen based on the FFT of the unfiltered signal, showing that the first three frequencies would be well inside the band, and higher frequencies would be well outside it.

The length of time modeled is selected to maximize the quantity of data used, while avoiding using data with a low signal to noise ratio (SNR). Because the magnitude of the response decays, the SNR decreases over approximately 0.75 seconds from about 25 to about 15. By utilizing data in the time domain all available information, including frequency, magnitude, and phase of the response, is used.

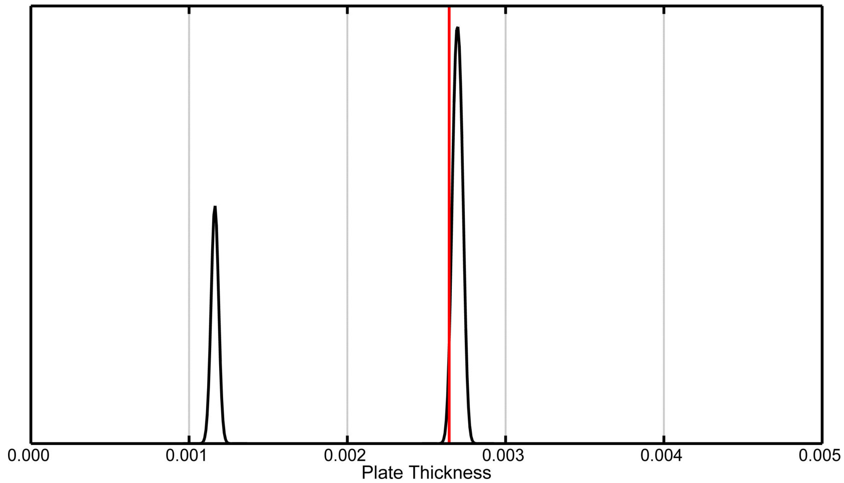

Figure 3: The probability density function of the non-dimensionalized plate thickness based on the recorded data. True value is shown as a solid vertical line.

5 RESULTS

The probability density function calculated for the plate thickness parameter is shown in Figure 3. The most likely value of thickness is 0.002645, which is 1.7% larger than the directly measured thickness. This is likely sufficiently accurate for industrial purposes. It is clear that there are two distinct peaks in the result. This is likely due to the fact that a change in thickness has a similar effect on the free response to a change in the edge spring constant parameter. It is important to note that the peak near the correct value about twice as tall as that at the spurious value, and is thus shown to be the more likely answer. Had a genetic algorithm been applied to this problem, only the location of the taller peak would have been found, and the user would remain ignorant of the possibility of a thinner plate/higher edge stiffness combination. In generating this result, a non-informative uniform prior was used for all parameters. It is likely that using information about length of service and environmental conditions, a more descriptive prior could be selected. Such a change would likely increase the certainty in the correct answer, and this ability is one of the main benefits of Bayesian methods.

6 CONCLUSIONS

This work demonstrates the potential that a Bayesian parameter identification methodology has for identifying key geometric features from an experimental system. In this case, determining the effective thickness of a plate provides a means for assessing the degree of corrosion damage that has occurred in a plate structure. The results show that the methodology was able to identify the correct plate thickness with a reasonable degree of certainty.

It is expected that this work will be expanded to consider a plate that has undergone actual corrosion damage. Using this pop-based Bayesian method, an effective thickness estimate (confirmed by alternative tests) will be used to assess the remain-

ing life of the plate.

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